

## 6-6: SYSTEMS OF LINEAR INEQUALITIES

Lesson Objectives:

- Solve systems of linear inequalities by graphing
- Model real-world situations using systems of linear inequalities

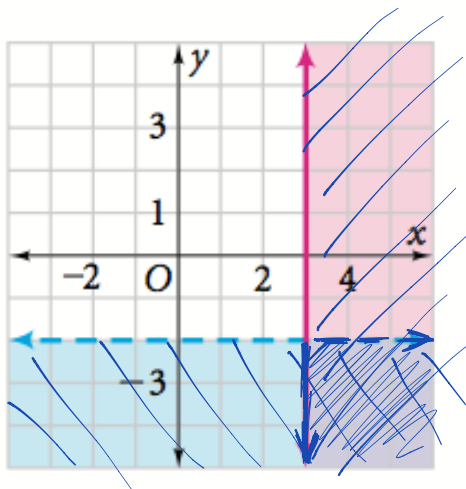
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### Solving Systems of Linear Inequalities by Graphing

Two or more linear inequalities together form a **system of linear inequalities**. The system below describes the lavender-shaded region of the graph. Notice that there are two boundary lines.

#### System of Linear Inequalities

$$\begin{cases} x \geq 3 \\ y < -2 \end{cases}$$



#### EXAMPLE 1: GRAPHING A SYSTEM OF INEQUALITIES

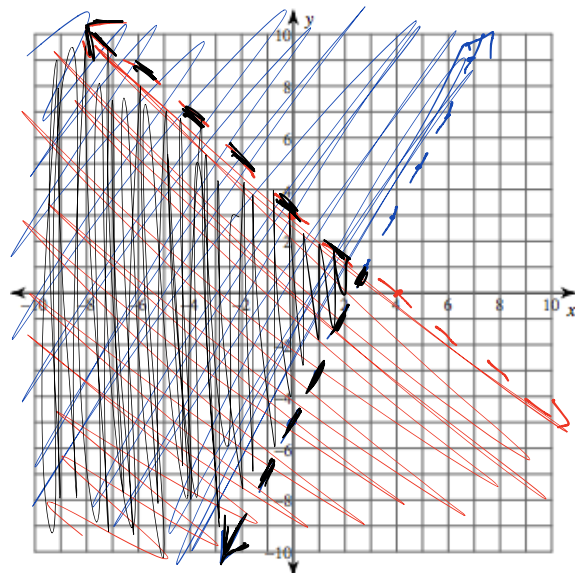
1. Solve by graphing.

$$\begin{aligned} 0 &> 2(6) - 5? \\ 0 &> 0 - 5? \\ 0 &> -5? \\ \text{yes} \end{aligned}$$

$$\begin{cases} y > 2x - 5 \\ 3x + 4y < 12 \end{cases}$$

$$\begin{array}{r} x \ 14 \\ 0 \ 3 \\ 4 \ 0 \end{array}$$

$$\begin{aligned} 0 &< 12? \\ \text{yes} \end{aligned}$$



Solve by graphing.

$$2. y < 2x - 3$$

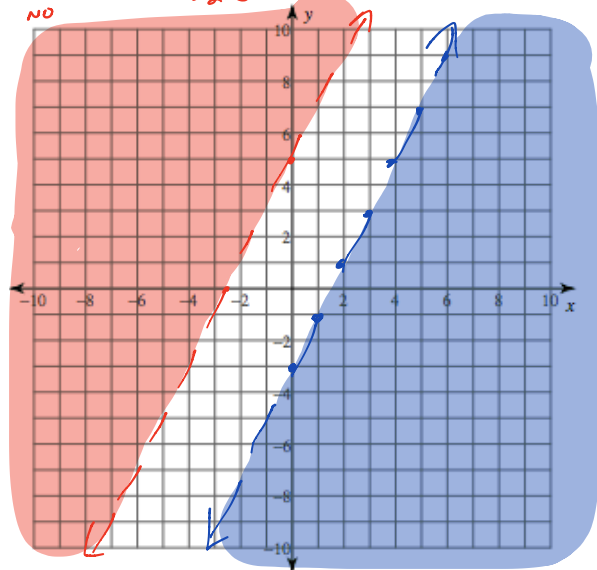
$$-2x + y > 5$$

$0 > 5?$   
no

$$\begin{array}{r} x \ 4 \\ 0 \ 5 \\ \hline -2 \frac{1}{2} \ 0 \end{array}$$

$0 < 2(0) - 3?$   
 $0 < 0 - 3?$   
 $0 < -3?$   
no

NO  
SOLUTION



$$\begin{array}{r} x \ 4 \\ 0 \ 4 \\ 5 \frac{1}{3} \ 0 \end{array}$$

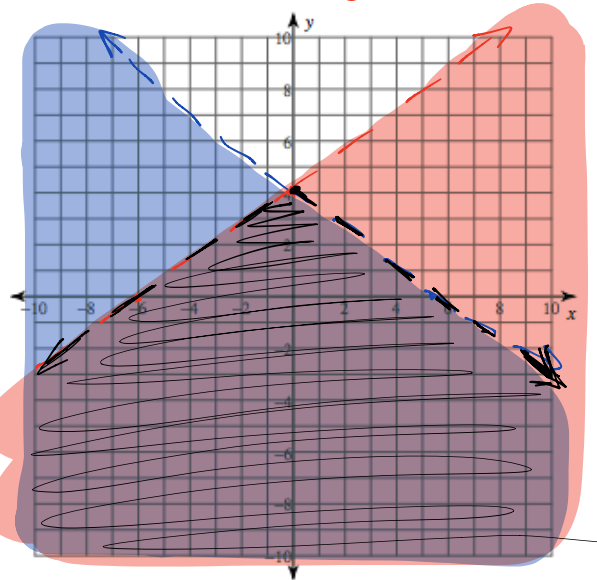
$0 < 32?$   
✓

$$3. 6x + 8y < 32$$

$$-4x + 6y < 24$$

$$\begin{array}{r} x \ 4 \\ 0 \ 4 \\ -6 \ 0 \end{array}$$

$0 < 24?$   
✓



## EXAMPLE 2: WRITING A SYSTEM OF INEQUALITIES FROM A GRAPH

You can combine your knowledge of linear equations with your knowledge of inequalities to describe a graph using a system of inequalities.

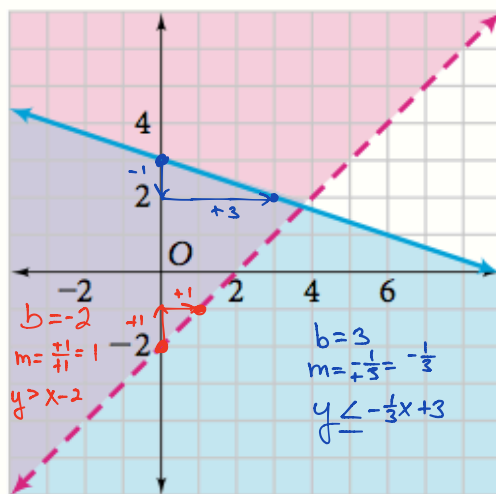
### 2 EXAMPLE Writing a System of Inequalities From a Graph

Write a system of linear inequalities from each shaded region below.

red region

boundary:  $y = x - 2$

The region lies above the boundary line, so the inequality is  $y > x - 2$ .



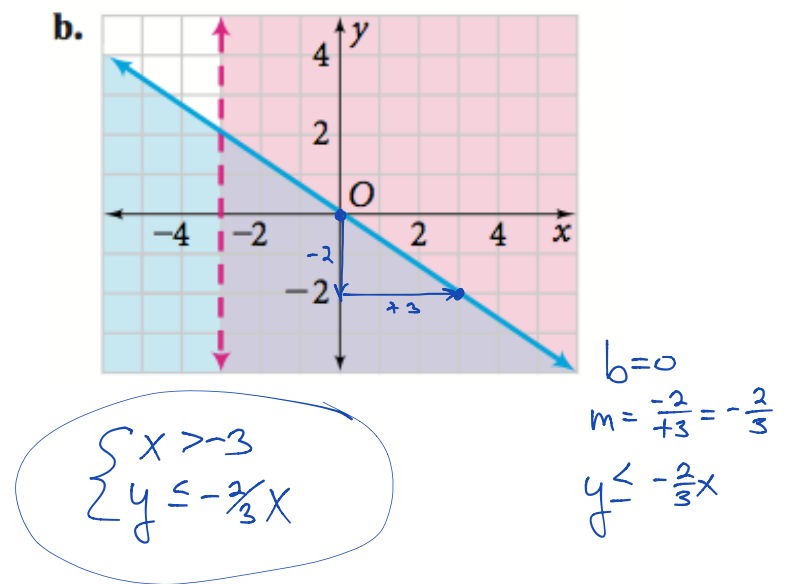
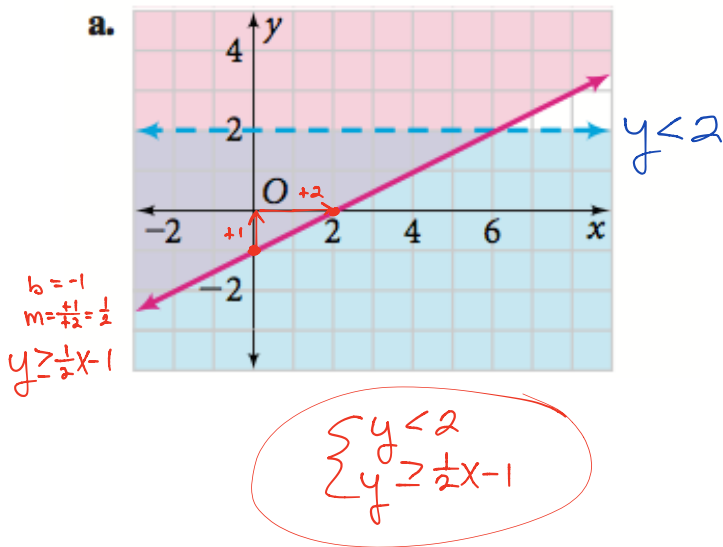
blue region

boundary:  $y = -\frac{1}{3}x + 3$

The region includes the boundary line and the points lying below the boundary line, so the inequality is  $y \leq -\frac{1}{3}x + 3$ .

system for the lavender region: 
$$\begin{cases} y > x - 2 \\ y \leq -\frac{1}{3}x + 3 \end{cases}$$

4. Write a system of inequalities for each of the following graphs.



You can model some real-world situations by graphing linear inequalities. When you graph real-world situations, you often need to plan how you will scale each axis. Use the values for the x- and y-intercepts to determine your scale.

## 2 Writing and Using Systems of Linear Inequalities

### EXAMPLE 3: REAL-WORLD PROBLEM SOLVING

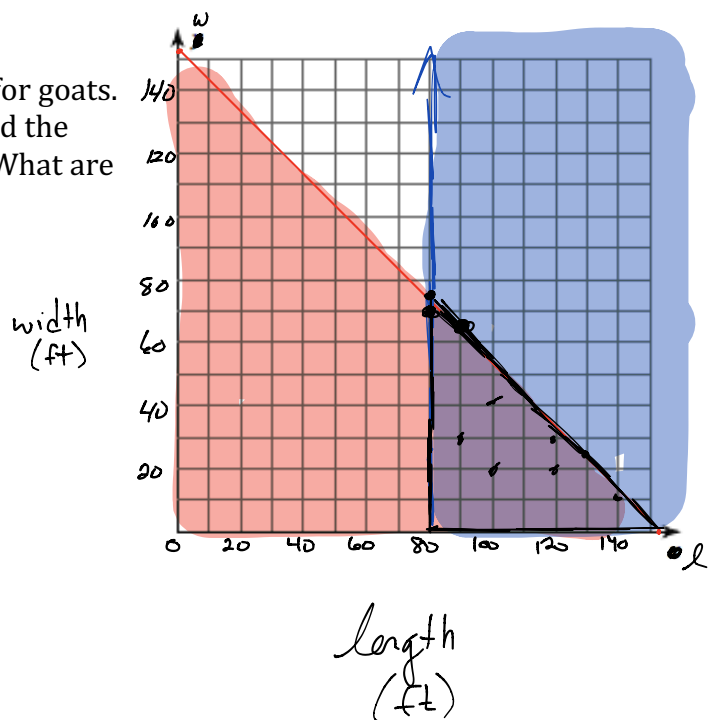
5. A zoo keeper wants to fence a rectangular habitat for goats. The length of the habitat should be at least 80 feet, and the distance around it should be no more than 310 feet. What are the possible dimensions of the habitat?

$$\begin{aligned} 2(80 \text{ ft}) + 2(70 \text{ ft}) &= 300 \\ 2(90) + 2(65) &= 310 \\ 2(80) + 2(75) &= 310 \end{aligned}$$

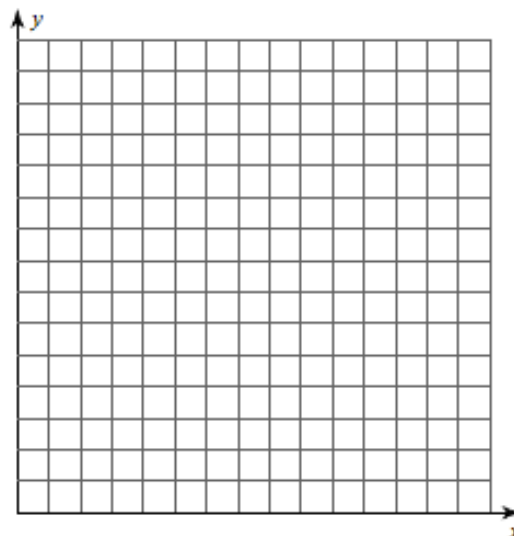
$$\begin{cases} 2l + 2w \leq 310 \\ l \geq 80 \end{cases}$$

Handwritten calculations for the system of inequalities:

$$\begin{array}{r} l \quad w \\ 2l + 2w \leq 310 \\ 0 \quad 155 \\ \hline 155 \quad 0 \end{array}$$



6. Suppose you want to fence a rectangular plot. You want the length of the garden to be at least 50 feet and the perimeter to be no more than 140 feet. Show all of the possible dimensions of the garden.



7. Suppose you need \$2.40 in postage to mail a package to a friend. You have 9 stamps, some 20 cents and some 34 cents. How many of each do you need to mail the package?

$$3 \text{ } 20¢ + 6 \text{ } 34¢ = 2.64$$

$$4 \text{ } 20¢ + 5 \text{ } 34¢ = 2.50$$

$$1 \text{ } 20¢ + 7 \text{ } 34¢ = 2.58$$

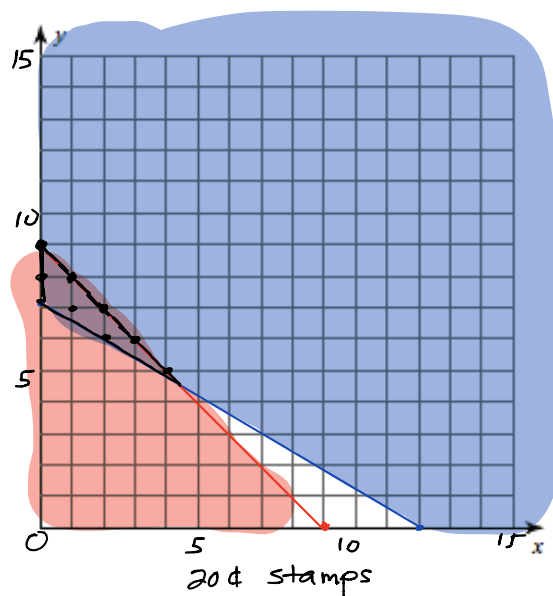
34¢ stamps

Let  $x = 20¢ \text{ stamps}$

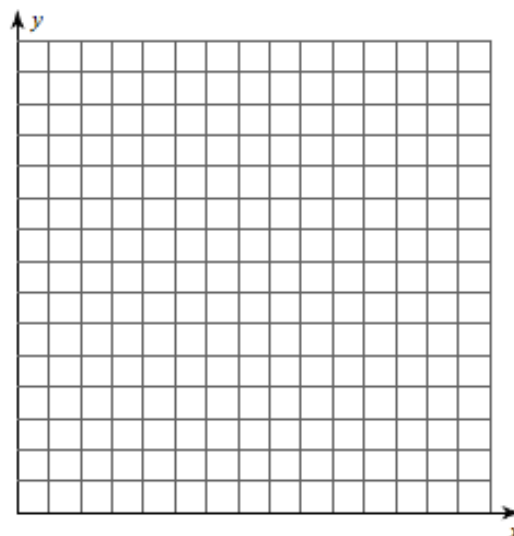
$y = 34¢ \text{ stamps}$

$$\begin{cases} 20x + 34y \geq 240 \\ x + y \leq 9 \end{cases}$$

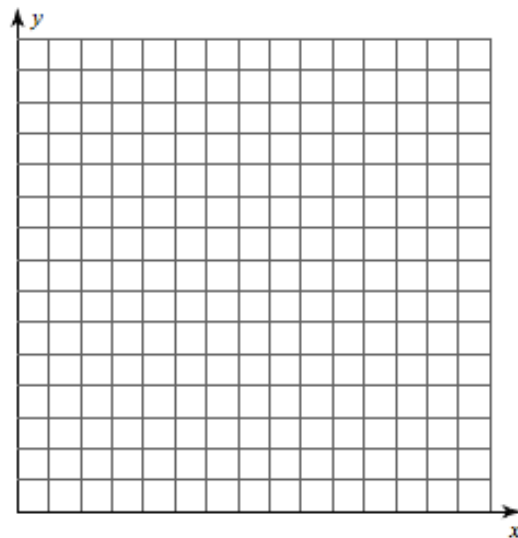
$$\begin{array}{r} x/y \\ 0/7.05 \\ 12/0 \end{array} \quad \begin{array}{r} x/y \\ 0/9 \\ 9/0 \end{array}$$



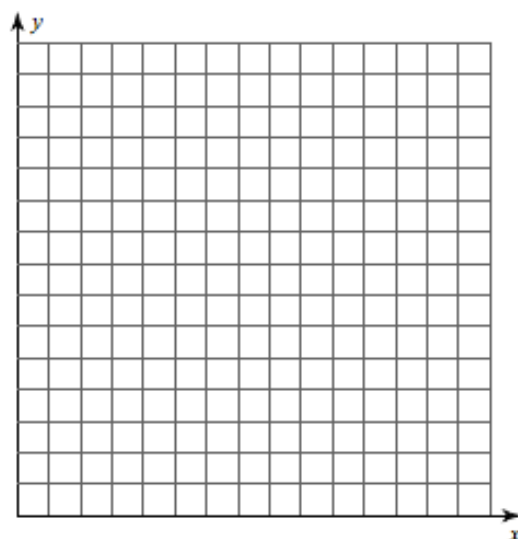
8. Suppose you intend to spend no more than \$60 buying books. Hardback books cost \$12 and paperbacks cost \$5. How many books of each type can you buy?



9. In basketball you score 2 points for a field goal and 1 point for a free throw. Suppose that you have scored at least 6 points in every game this season, and have a season high score of 15 points in one game. How many field goals and free throws could have made in any one game?



10. Suppose you need to use at least \$1.00 worth of stamps to mail a package. You have as many 3 cent stamps as you need, but only four 32 cent stamps. How many of each stamp can you use? Write two possible combinations that you could use.



11. A grandmother wants to spend at least \$40 but no more than \$60 on school clothes for her grandson. T-shirts sell for \$10 and pants sell for \$20. How many T-shirts and pants could she buy? Write two possible combinations of T-shirts and pants.

$$\begin{array}{rclcl}
 2 & \text{T-shirts} & + & 2 & \text{pants} & = & 60 \\
 4 & " & + & 1 & " & = & 60 \\
 1 & " & + & 2 & " & = & 50 \\
 3 & " & + & 1 & " & = & 50
 \end{array}$$

pants

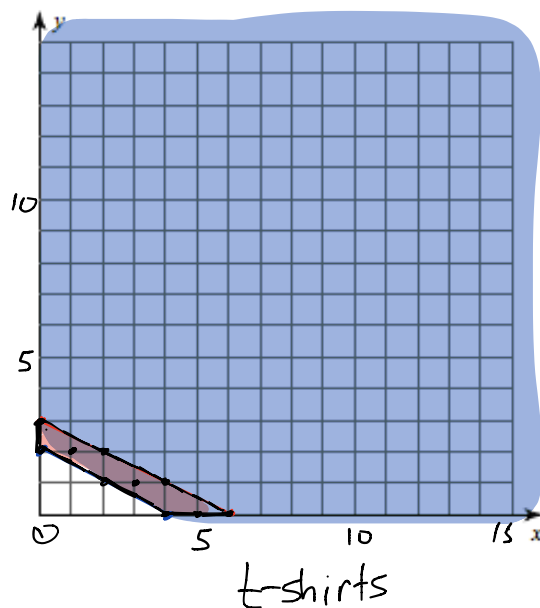
let  $x = \text{T-shirts}$   
 $y = \text{pants}$

$$10x + 20y \geq 40$$

$$10x + 20y \leq 60$$

$$\begin{array}{r}
 \times 14 \\
 0 \overline{) 2} \\
 4 \overline{) 0}
 \end{array}$$

$$\begin{array}{r}
 \times 14 \\
 0 \overline{) 3} \\
 6 \overline{) 0}
 \end{array}$$



**Is the given ordered pair a solution of the system of inequalities?**

(4,10)

(-2,40)

1.  $9x - y \geq 23$

2.  $y > -13x + 29$

$5x + 0.2y \geq 20$

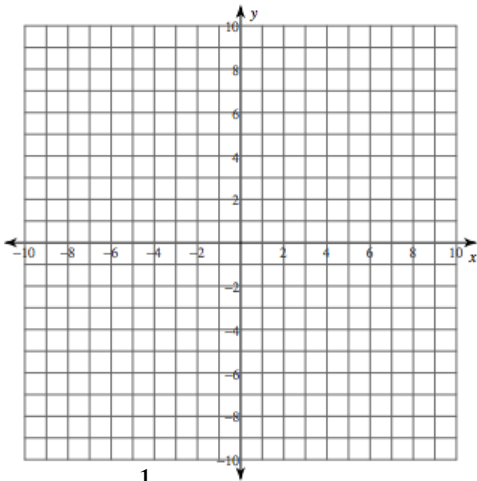
$y \leq 9x + 11$

**Solve each system of inequalities by graphing.**

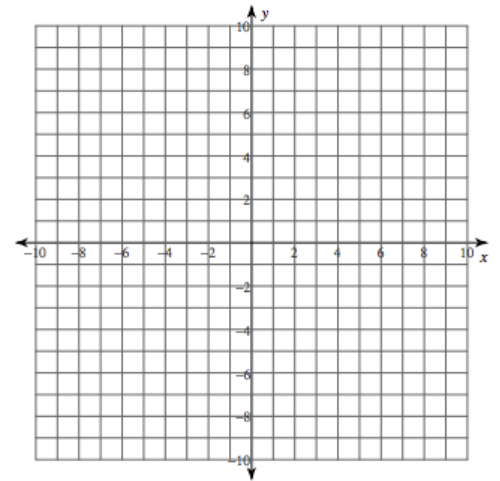
3.  $y < 2x + 4$

$-3x - 2y \geq 6$

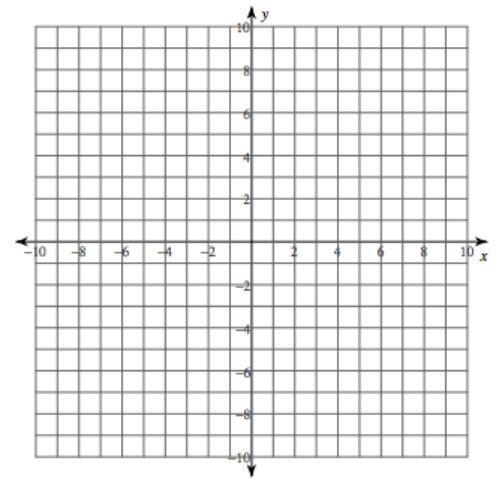
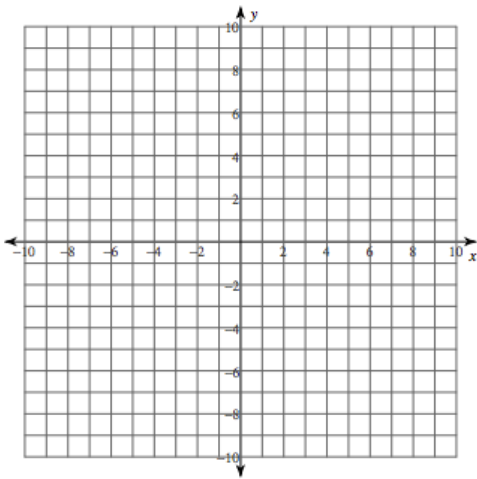
4.  $y \leq 2x - 3$   
 $y > 5$



5.  $y \leq -\frac{1}{3}x + 7$   
 $y \geq -x + 1$

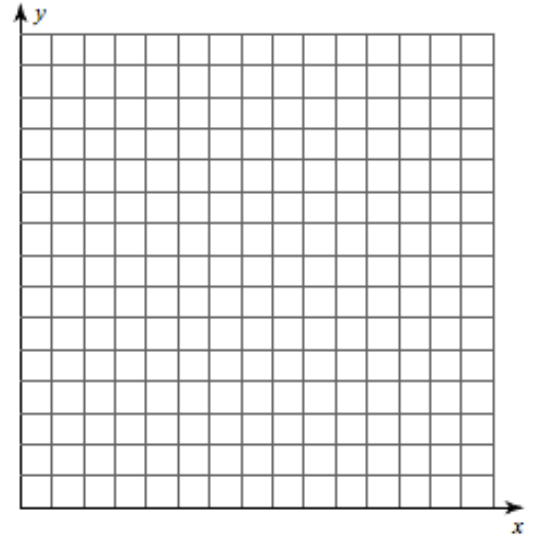


6.  $8x + 4y \geq 10$   
 $3x - 6y > 12$



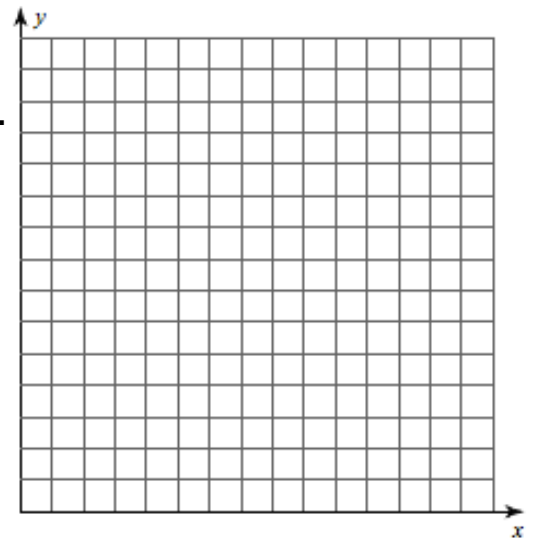
7. Suppose you want to fence a rectangular area for your dog. You will use the house as one of the four sides. Since the house is 40 ft wide, the length  $l$  needs to be no more than 40 ft. You plan to use at least 150 ft of fencing.

- Write a system of inequalities that describes the situation.
- Graph the system
- Find two possible dimensions for the fence.

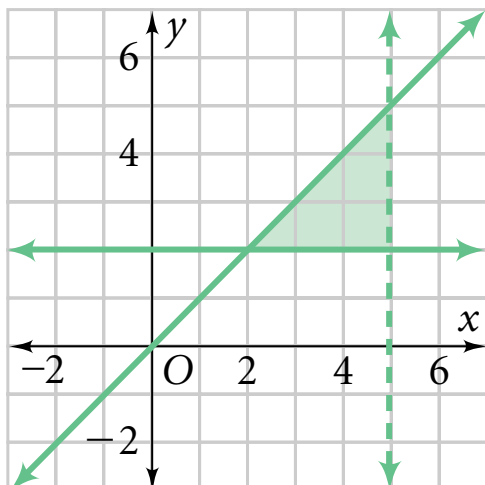


8. Suppose you have a job in an ice cream shop that pays \$8 per hour. You also have a babysitting job that pays \$10 per hour. You want to earn at least \$100 per week but would like to work no more than 15 hours per week.

- Write a system of inequalities that describes the situation.
- Graph the system.
- Find two possible combinations of hours that you could work.



9. Write a system of inequalities for the graph.



10. A jeweler plans to produce a ring made of silver and gold. The price of gold is approximately \$40 per gram. The price of silver is approximately \$0.50 per gram. She considers the following in deciding how much gold and silver to use in the ring.

- The total mass must be more than 10 grams, but less than 20 grams.
- The ring must contain at least 4 grams of gold.
- The total cost of the gold and silver must be less than \$500.

Let  $s$  = the mass of silver in grams and  $g$  = the mass of gold in grams.

- Write and graph the four inequalities that describe this situation.
- For one solution, find the mass of the ring and the cost of the gold and silver.

$$\begin{cases} s + g > 10 \\ s + g < 20 \\ g \geq 4 \\ .5s + 40g < 500 \end{cases}$$

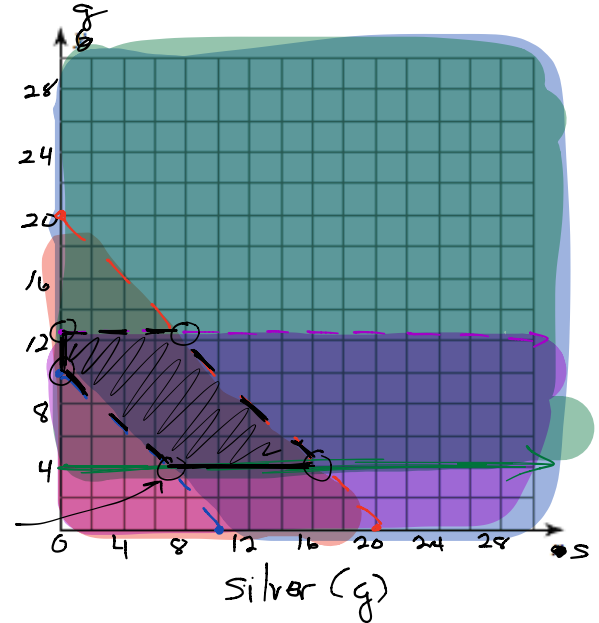
Handwritten calculations for solving the system:

$$\begin{array}{r} s \quad g \\ 0 \quad 10 \\ 10 \quad 0 \end{array}$$

$$\begin{array}{r} s \quad g \\ 0 \quad 20 \\ 20 \quad 0 \end{array}$$

$$\begin{array}{r} s \quad g \\ 0 \quad 12.5 \\ 1000 \quad 0 \end{array}$$

gold  
(g)



11. A drum maker sells two sizes of frame drums. A 14-in. drum sells for \$180 and an 18-in. drum sells for \$240. He is trying to decide how many drums to build and considers the following:

- He wants to produce and sell at least \$2700 worth of drums.
- He has materials to make no more than 17 drums.
- He plans to make more 14-in. drums than 18-in. drums.
- He wants to make at least four 18-in. drums.

- Write and graph the four inequalities that describe the situation
- Give one possible solution to the system.

